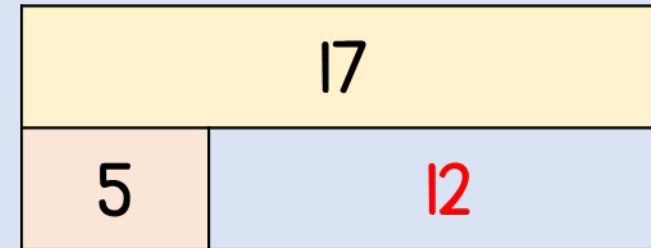


# Maths Curriculum Meeting for Parents

- 1) Use  $<$ ,  $>$  or  $=$  to compare the number sentences.

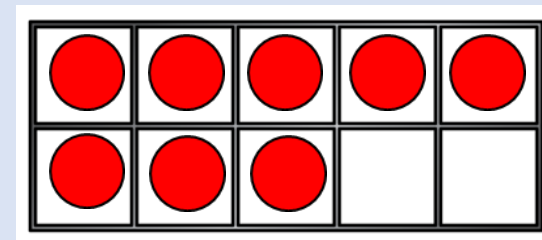
$$5 + 6 \quad \textcircled{=} \quad 6 + 5$$

- 2) Complete the bar model.

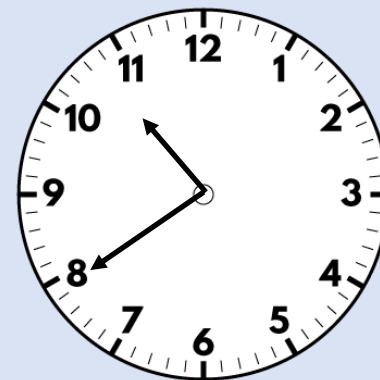


- 3) Add together 7 and 6 13

- 4) What number is shown?



8

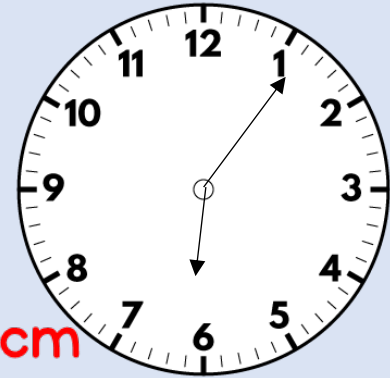


1) Work out  $14 \times 4$  **56**

2) If  $3 \times 8 = 24$ , what is  $80 \times 3$ ? **240**

3) What is  $96 \div 8$ ? **12**

4) Find the difference between 182 and 175 **7**



1) If a rectangle has area  $16 \text{ cm}^2$ , what could the side lengths of the rectangle be?

e.g. 8 cm and 2 cm, 16 cm and 1 cm, 4cm and 4cm

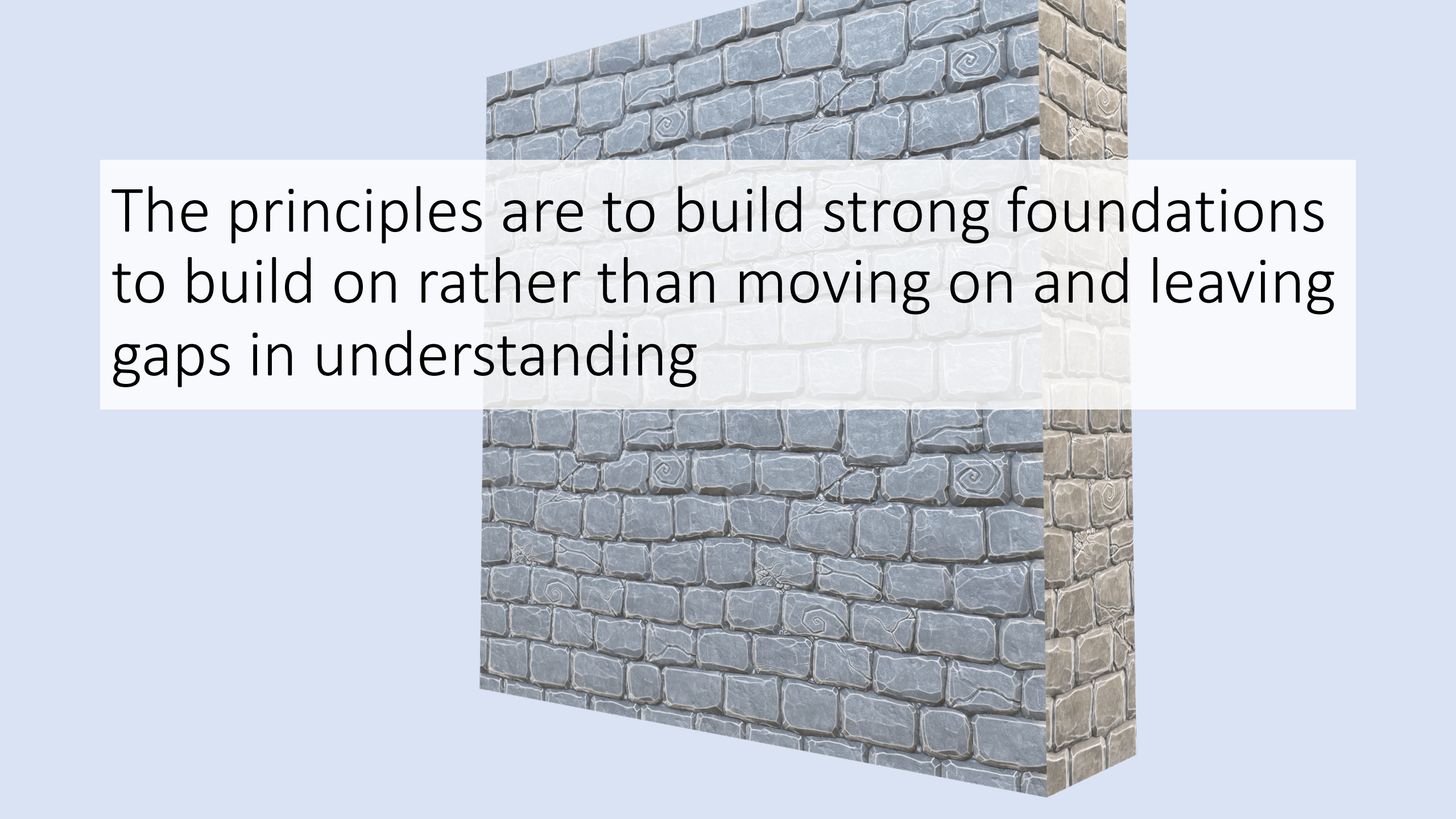
2) How many cm are there in 3.7 m? 370

3) Solve the equation  $47 = y - 5$   $y = 52$

4) Work out  $\frac{7}{8}$  of 64 56



The curriculum  
implemented in 2014 is  
based on mastery

The image features a 3D-rendered stone wall that recedes into the distance, creating a sense of depth. The wall is composed of dark grey, rectangular stones with visible mortar joints. A central portion of the wall is covered by a translucent white rectangular box. Inside this box, the text "The principles are to build strong foundations to build on rather than moving on and leaving gaps in understanding" is written in a black, sans-serif font. The background of the entire image is a solid light blue color.

The principles are to build strong foundations  
to build on rather than moving on and leaving  
gaps in understanding



# Mastery

**Mastery** is based on the idea of children

- Not moving on until they are secure in their understanding of a particular concept.
- The whole class is taught the same thing, at the same time, with children learning at an appropriate level through support and enrichment.

# Exercise 1


Take the following calculations and complete:

$$2345 + 2764 \quad 12876 + 23456 \quad 30987564 + 234572$$

$$\begin{array}{r} 1234568321 \\ +2874650534 \end{array}$$

$$\begin{array}{r} 3000000000 \\ - 1000000001 \end{array}$$

Does this prove that the children are good at maths?



# What are the key components of mastery?

---

Fluency

Reasoning

Problem  
Solving



# Exercise 2: Questions that a teacher may ask

Calculate  $123 + 86$

How do you know that you have the correct answer?

What strategy could you use to make the calculation easier?

Now show me at least 2 other ways to calculate this:

Is  $123 + 86$  the same as  $126 + 83$ ?

How do you know? Prove it

The mastery concept underpins  
our calculation policy as we  
want children to fully  
understand number rather than  
carry out procedure.



# Calculation Policy

The video below demonstrates what mastery might look like.

A sense of number: <https://vimeo.com/152139735>

The development of calculation is based on three main principles:

Concrete, Pictorial, Abstract

In that order, but they are inter-related so that the children build small building blocks within larger building blocks, which all lead to a secure understanding.

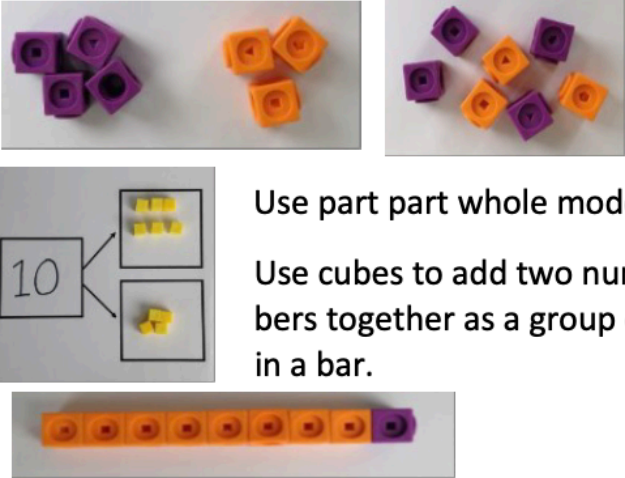
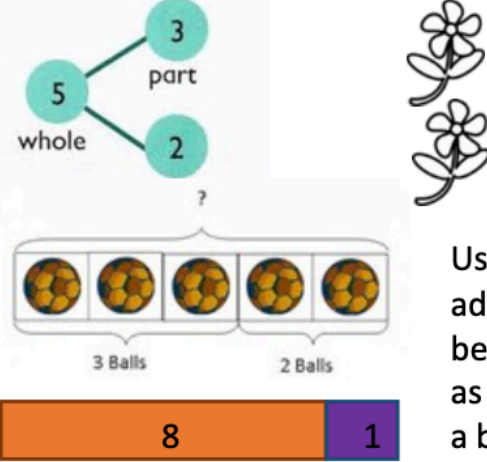

The calculation policy slides show progression in addition, subtraction, multiplication and division. These are some of the steps, not all.

Please note:

The long division examples on the policy show long division using division by 2. This is to show the method.

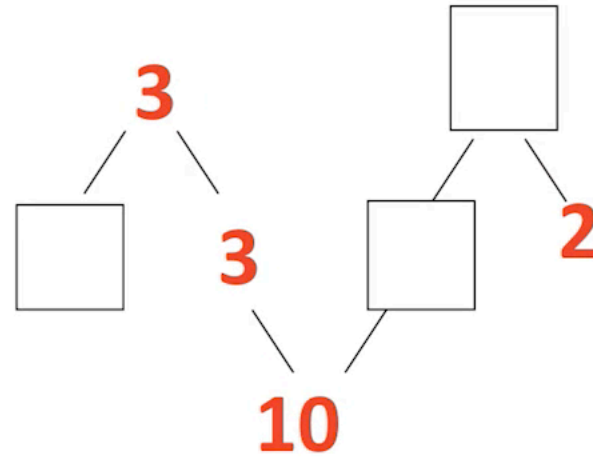
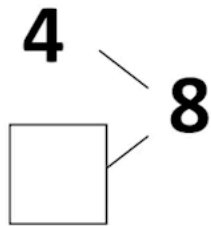
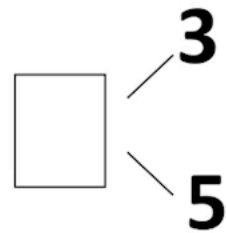
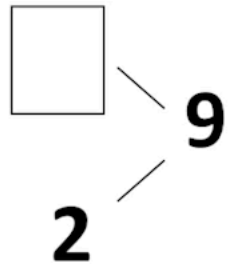
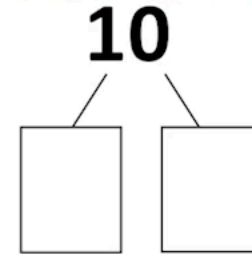
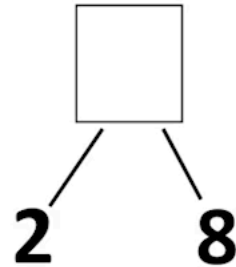
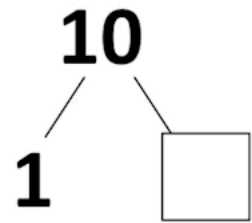
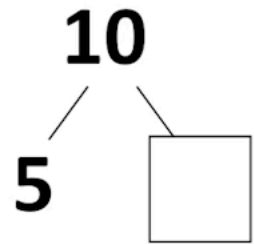
We would not expect our children to use long division when dividing by 2.

# Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p><math>4 + 3 = 7</math></p>  <p><math>10 = 6 + 4</math></p> <p>Use the part-part whole diagram as shown above to move into the abstract.</p>

Is  $9 + 1$  the same as  $1 + 9$ ?

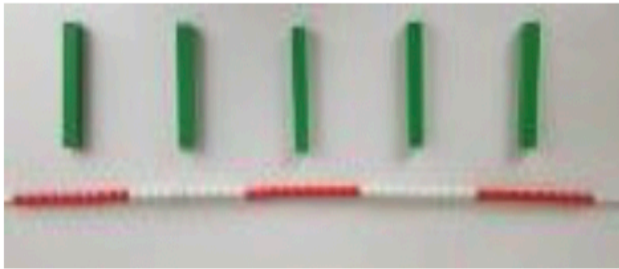
# This is what we call variation



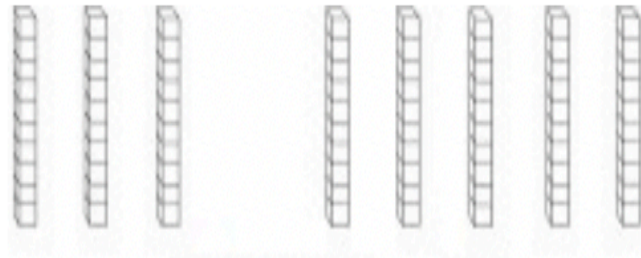
# Reinforcement through different representations:

Adding multiples of ten

$$50 = 30 + 20$$



Model using dienes and bead strings



$$3 \text{ tens} + 5 \text{ tens} = \underline{\quad} \text{ tens}$$

$$30 + 50 = \underline{\quad}$$

Use representations for base ten.

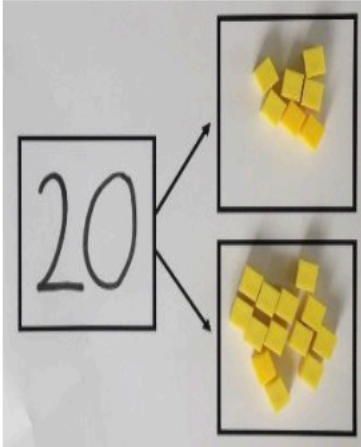
$$20 + 30 = 50$$

$$70 = 50 + 20$$

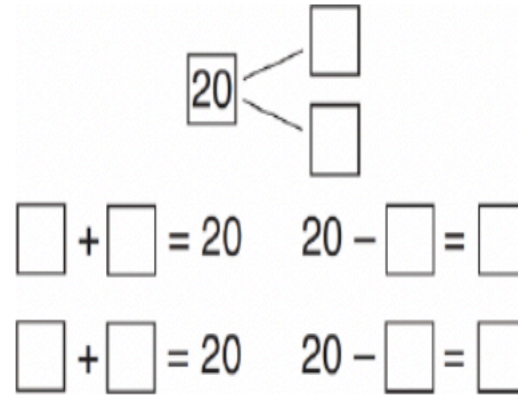
$$40 + \square = 60$$

Use known number  
facts

Part part whole



Children ex-  
plore ways of  
making num-  
bers within 20



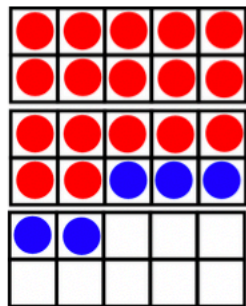
$$\square + 1 = 16$$

$$16 - 1 = \square$$

$$1 + \square = 16$$

$$16 - \square = 1$$

Add a two digit  
number and ones



$$17 + 5 = 22$$

Use ten frame to  
make 'magic ten

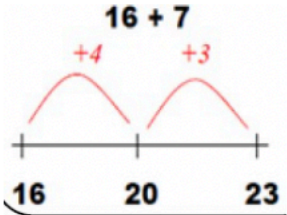
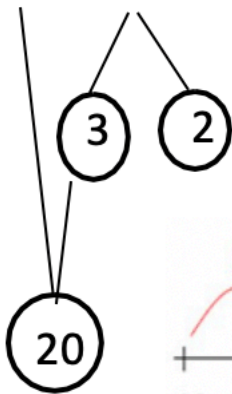
Children explore the pattern.

$$17 + 5 = 22$$

$$27 + 5 = 32$$

Use part  
part whole  
and number  
line to  
model.

$$17 + 5 = 22$$



$$17 + 5 = 22$$

Explore related facts

$$17 + 5 = 22$$

$$5 + 17 = 22$$

$$22 - 17 = 5$$

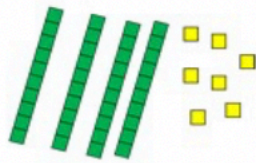
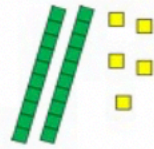
$$22 - 5 = 17$$

22	
17	5

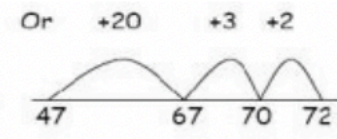
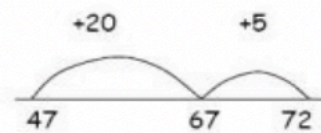


# Practise

Add two 2-digit numbers



Model using dienes, place value counters and numicon



Use number line and bridge ten using part whole if necessary.

$$\begin{array}{c}
 25 + 47 \\
 \swarrow \quad \downarrow \quad \searrow \\
 20 + 5 \quad \quad 40 + 7
 \end{array}$$

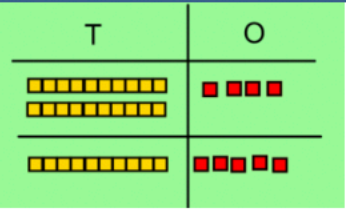
$$20 + 40 = 60$$

$$5 + 7 = 12$$

$$60 + 12 = 72$$

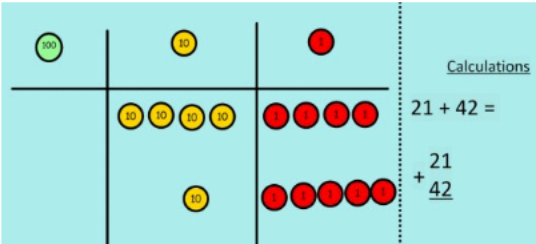
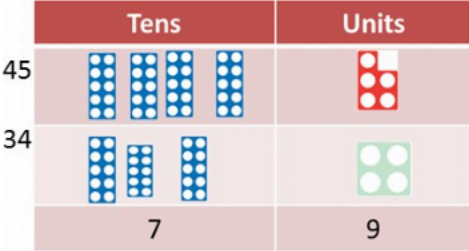
Column Addition—no regrouping (friendly numbers)

Add two or three 2 or 3-digit numbers.



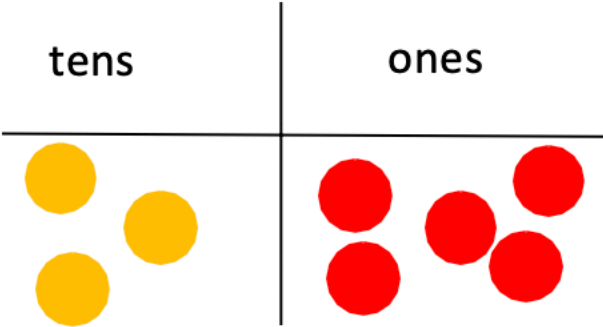
Model using  
Dienes or nu-  
micon

Add together the ones first, then the tens.



Move to using place value counters

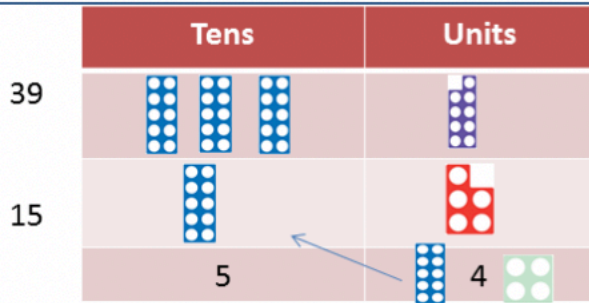
Children move to drawing the counters using a tens and one frame.



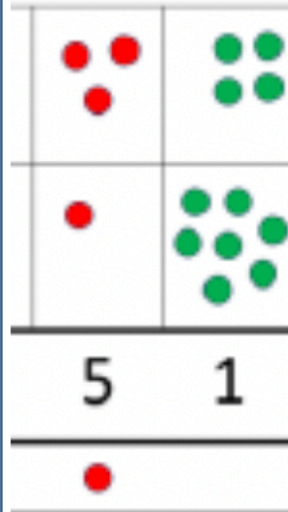
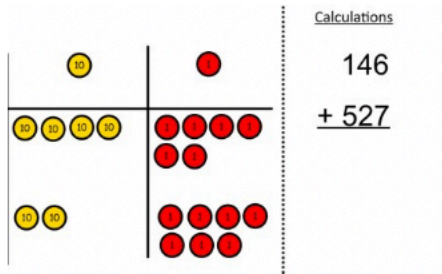
$$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$$

Add the ones first, then the tens, then the hundreds.

Column Addition with regrouping.



Exchange ten ones for a ten. Model using numicon and py counters.



Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line

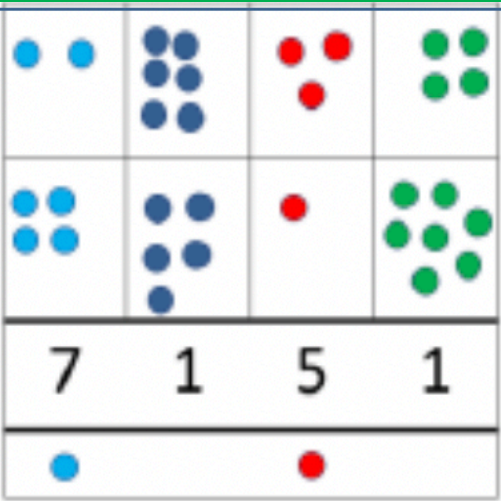
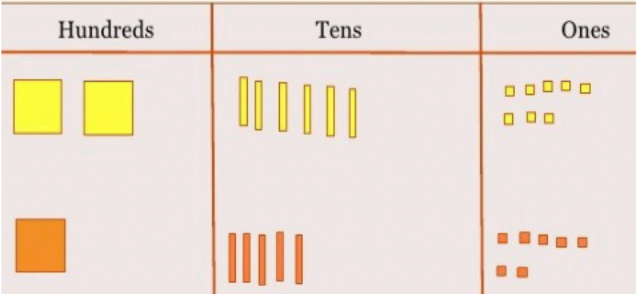
$$\begin{array}{r} 20 \\ 40 \\ 60 \end{array} + \begin{array}{r} 5 \\ 8 \\ 13 \end{array} = 73$$

Start by partitioning the numbers before formal column to show the exchange.

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$









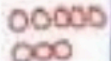

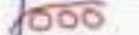


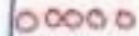
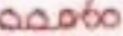
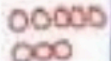

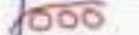


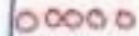
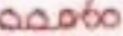




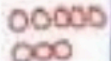

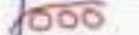


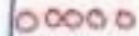
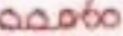
Y4—add numbers with up to 4 digits

Children continue to use ~~ones~~ or ~~pv~~ counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.



Draw representations using pv grid.

Continue from previous work to carry hundreds as well as tens.  
Relate to money and measures.

<p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p> <table><tr><td>tens</td><td>ones</td><td></td><td>tenths</td><td>hundredths</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table> <p>Introduce decimal place value counters and model exchange for addition.</p>	tens	ones		tenths	hundredths						<p><math>2.37 + 81.79</math></p> <table><tr><td>tens</td><td>ones</td><td>tenths</td><td>hundredths</td></tr><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr></table> <p>6</p>	tens	ones	tenths	hundredths									<p><math>72.8</math> <math>+ 54.6</math> <u><math>127.4</math></u> 11</p> <table><tr><td>£</td><td>2</td><td>3</td><td>.</td><td>5</td><td>9</td></tr><tr><td>+</td><td>£</td><td>7</td><td>.</td><td>5</td><td>5</td></tr><tr><td colspan="6"><hr/></td></tr><tr><td>£</td><td>3</td><td>1</td><td>.</td><td>1</td><td>4</td></tr></table>	£	2	3	.	5	9	+	£	7	.	5	5	<hr/>						£	3	1	.	1	4
tens	ones		tenths	hundredths																																													
																																																	
tens	ones	tenths	hundredths																																														
																																																	
																																																	
£	2	3	.	5	9																																												
+	£	7	.	5	5																																												
<hr/>																																																	
£	3	1	.	1	4																																												

Y6—add several numbers of increasing complexity

Including adding money, measure and decimals with different numbers of decimal points.

As Y5

As Y5

$$\begin{array}{r} 81,059 \\ 3,668 \\ 15,301 \\ + 20,551 \\ \hline 120,579 \\ \begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \end{array}$$

Insert zeros for place holders.

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \end{array}$$

# Subtraction

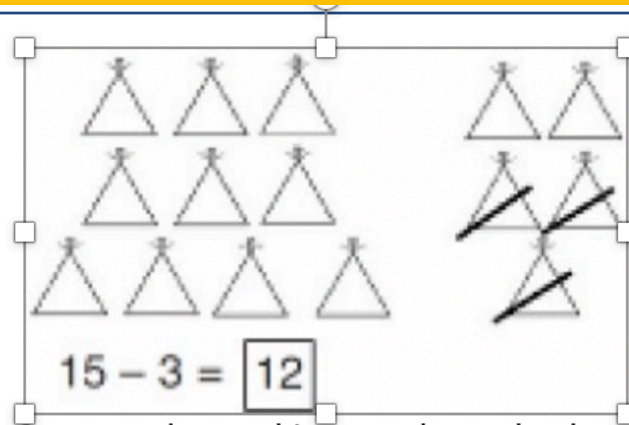
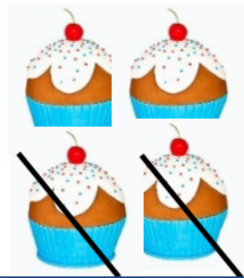
Taking away  
ones.

Use physical objects, counters, cubes etc  
to show how objects can be taken away.



$$6 - 4 = 2$$

$$4 - 2 = 2$$



Cross out drawn objects to show what has  
been taken away.

$$7 - 4 = 3$$

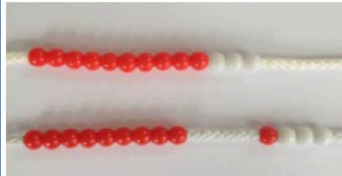
$$16 - 9 = 7$$



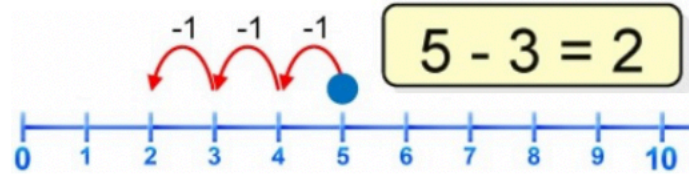
## Counting back



Move objects away from the group, counting backwards.



Move the beads along the bead string as you count backwards.

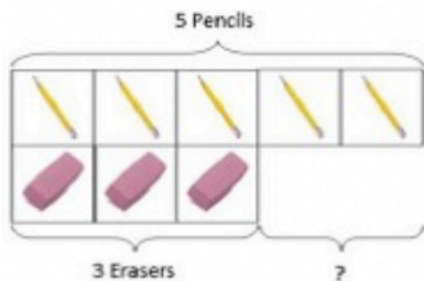
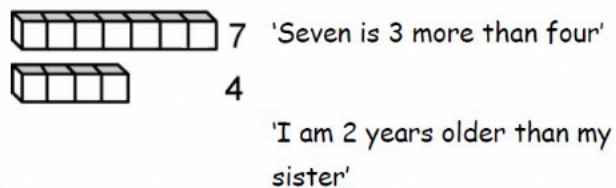


Count back in ones using a number line.

Put 13 in your head, count back 4. What number are you at?

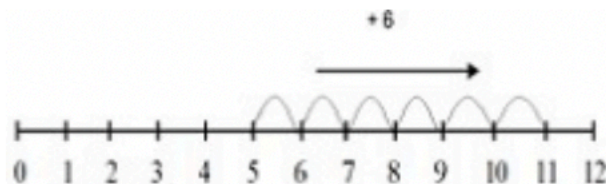
Find the  
Difference

Compare objects and amounts



Lay objects to represent bar model.

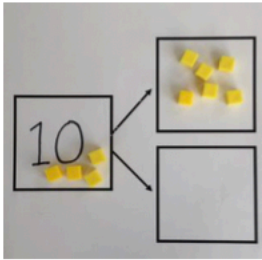
Count on using a number line to find the difference.



Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

**Represent and use  
number bonds and  
related subtraction  
facts within 20**

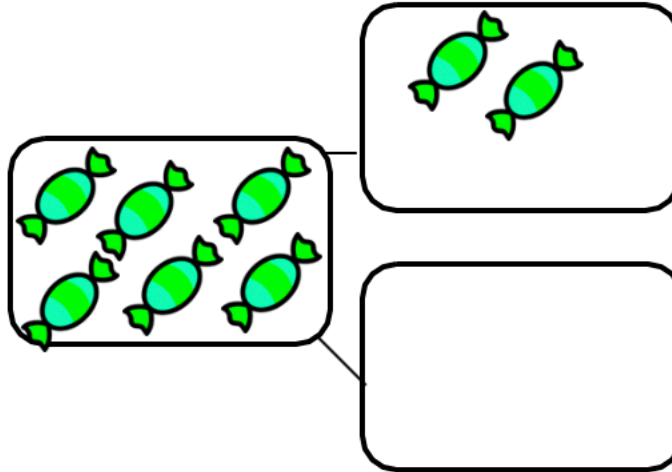
**Part Part Whole  
model**



Link to addition. Use  
PPW model to model  
the inverse.

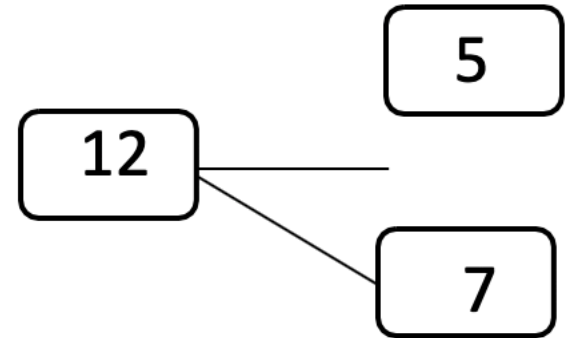
If 10 is the whole and 6 is one of the parts,  
what s the other part?

$$10 - 6 = 4$$



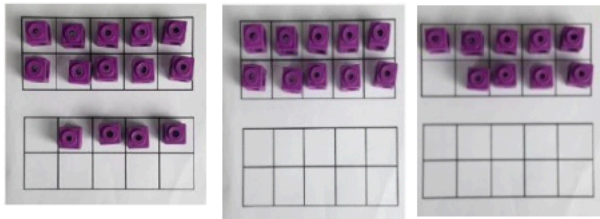
Use pictorial representations to show the part.

Move to using numbers within  
the part whole model.



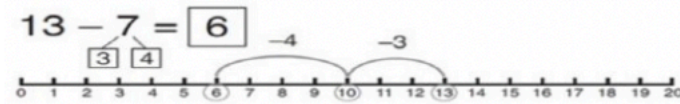
Make 10

$$14 - 9$$



Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.

$$13 - 7$$

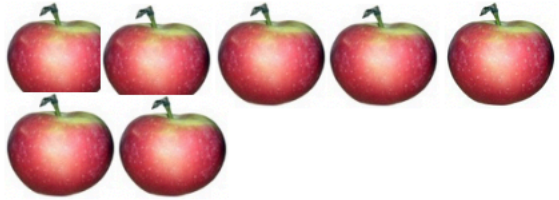


Jump back 3 first, then another 4. Use ten as the stopping point.

$$16 - 8$$

How many do we take off first to get to 10? How many left to take off?

Bar model



$$5 - 2 = 3$$



8

2

$$10 = 8 + 2$$

$$10 = 2 + 8$$

$$10 - 2 = 8$$

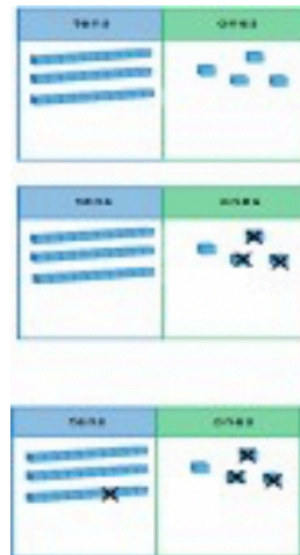
$$10 - 8 = 2$$

Partitioning to subtract without regrouping.

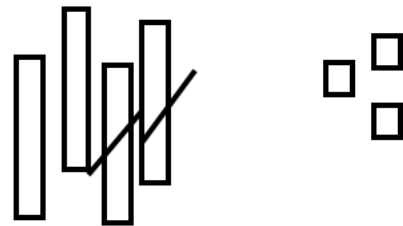
*'Friendly numbers'*

$$34 - 13 = 21$$

Use Dienes to show how to partition the number when subtracting without regrouping.



Children draw representations of Dienes and cross off.



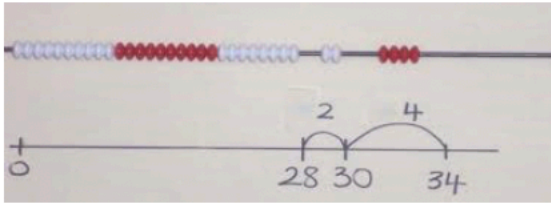
$$43 - 21 = 22$$

$$43 - 21 = 22$$

# Practise

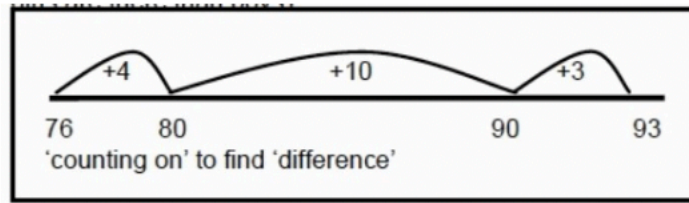
## Make ten strategy

*Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*



$$34 - 28$$

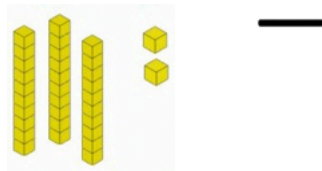
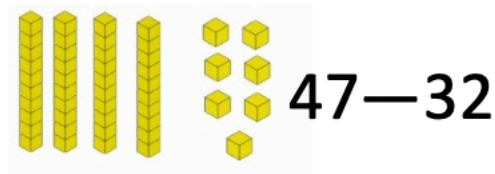
Use a bead bar or bead strings to model counting to next ten and the rest.



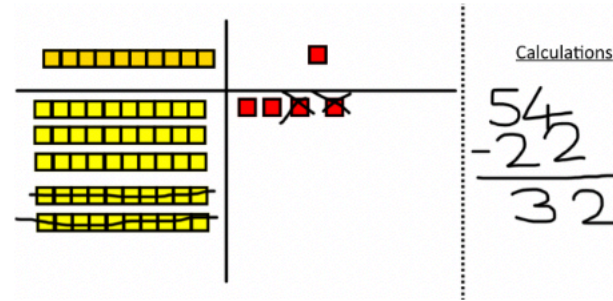
Use a number line to count on to next ten and then the rest.

$$93 - 76 = 17$$

Column subtraction  
without regrouping  
(friendly numbers)



Use base 10 or Numicon to model



Darw representations to support understanding

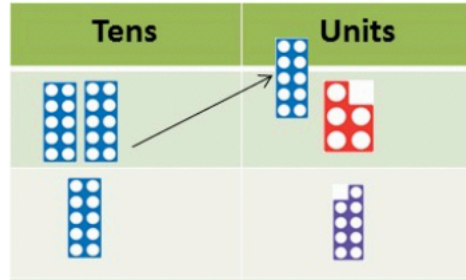
$$\begin{array}{r} 47 - 24 = 23 \\ \underline{40 + 7} \\ - \underline{20 + 4} \\ 20 + 3 \end{array}$$

Intermediate step may  
be needed to lead to  
clear subtraction under-  
standing.

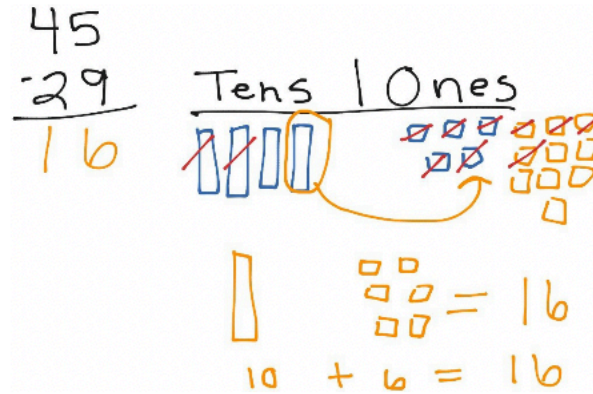
$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$



## Column subtraction with regrouping



Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.



Children may draw base ten or PV counters and cross off.

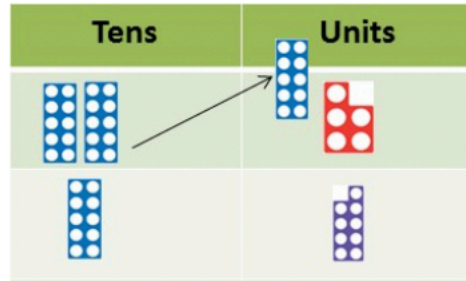
$$\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{ccc} \text{H} & \text{T} & \text{U} \\ 800 & 30 & 6 \\ - 200 & 50 & 4 \\ \hline 500 & 80 & 2 \end{array} \end{array}$$

Begin by parti-  
tioning into pv  
columns

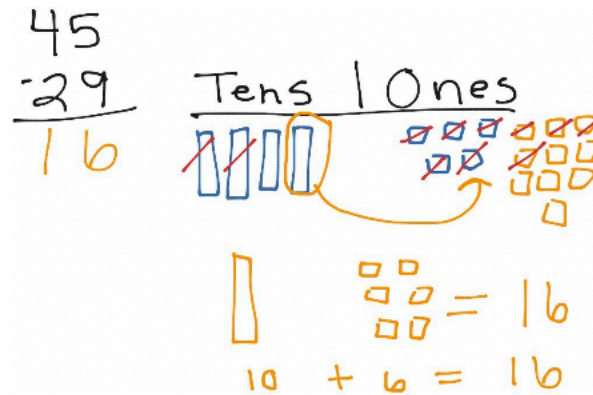
$$\begin{array}{r} 728 - 582 = 146 \\ \begin{array}{ccc} \text{H} & \text{T} & \text{U} \\ 7 & 2 & 8 \\ - 5 & 8 & 2 \\ \hline 1 & 4 & 6 \end{array} \end{array}$$

Then move to  
formal method.

## Column subtraction with regrouping



Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.



Children may draw base ten or PV counters and cross off.

$$\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{ccc} \text{H} & \text{T} & \text{U} \\ \cancel{8}00 & 130 & 6 \\ - 200 & 50 & 4 \\ \hline 500 & 80 & 2 \end{array} \end{array}$$

Begin by parti-  
tioning into pv  
columns

$$\begin{array}{r} 728 - 582 = 146 \\ \begin{array}{ccc} \text{H} & \text{T} & \text{U} \\ \cancel{7} & 2 & 8 \\ 5 & 8 & 2 \\ \hline 1 & 4 & 6 \end{array} \end{array}$$

Then move to  
formal method.

<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>As Year 4</p>	<p>Children to draw <u>py</u> counters and show their exchange—see Y3</p>	<div data-bbox="1854 482 2267 706"> </div> <p>Use zeros for place-holders.</p>
--	------------------	---	--

Year 6—Subtract  
with increasingly  
large and more  
complex numbers  
and decimal values.

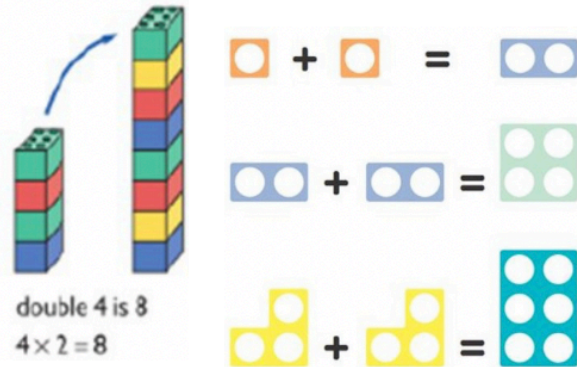
$$\begin{array}{r}
 \cancel{8}^9 \cancel{10}^9, 699 \\
 - \quad 89,949 \\
 \hline
 60,750
 \end{array}$$

$$\begin{array}{r}
 \cancel{10}^5 \cdot \cancel{14}^1 9 \text{ kg} \\
 - \quad 36 \cdot 08 \text{ kg} \\
 \hline
 69 \cdot 339 \text{ kg}
 \end{array}$$

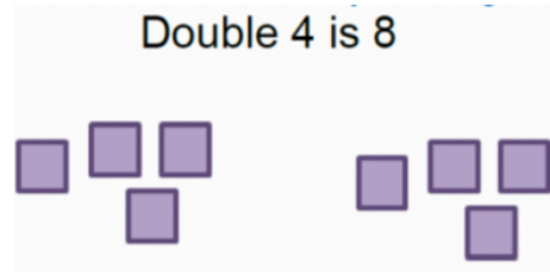
# Multiplication

## Doubling

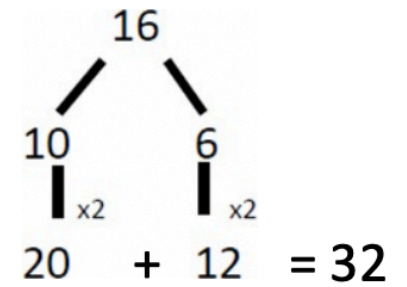
Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling



Draw pictures to show how to double numbers

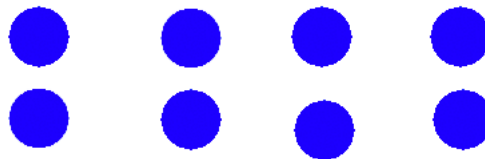


Partition a number and then double each part before recombining it back together.

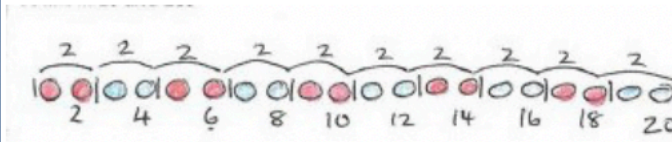


Counting in multi-  
ples

Count the groups as children are skip counting, children may use their fingers as they are skip counting.



Children make representations to show counting in multiples.



Count in multiples of a number aloud.

Write sequences with multiples of num-  
bers.


2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30

Making equal  
groups and  
counting the total



Use manipulatives to create equal groups.

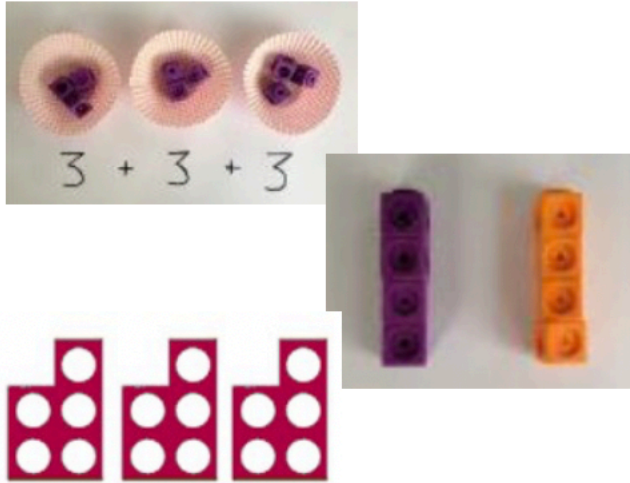
Draw  to show  $2 \times 3 = 6$

Draw and make representations

$$2 \times 4 = 8$$



## Repeated addition



Use different objects to add equal groups

Use pictorial including number lines to solve problems.

There are 3 sweets in one bag.  
How many sweets are in 5 bags altogether?

$$\begin{array}{c} \bigcirc \\ \bullet \quad \bullet \quad \bullet \end{array} \quad \begin{array}{c} \bigcirc \\ \bullet \quad \bullet \quad \bullet \end{array} \quad 3+3+3+3+3 \\ = 15$$



Write addition sentences to describe objects and pictures.

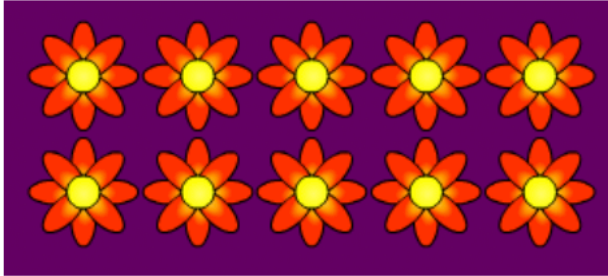


$$2+2+2+2+2=10$$



Understanding ar-  
rays

Use objects laid out in arrays to find the an-  
swers to 2 lots 5, 3 lots of 2 etc.



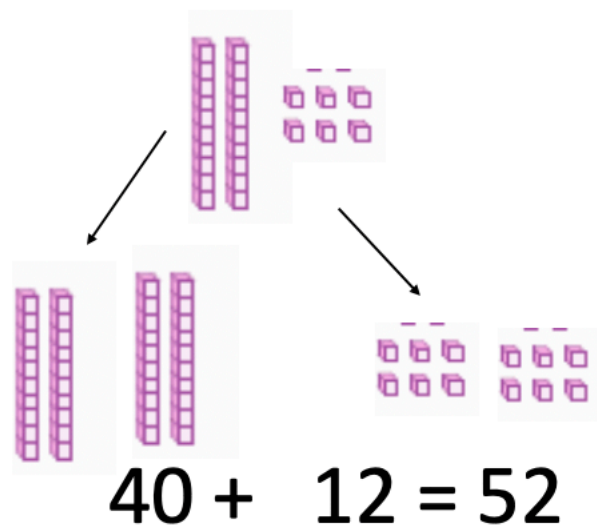
Draw representations of arrays to show under-  
standing

$$3 \times 2 = 6$$

$$2 \times 5 = 10$$

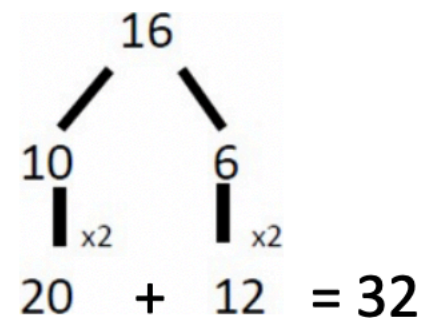
## Doubling

Model doubling using dienes and PV counters.



Draw pictures and representations to show how to double numbers

Partition a number and then double each part before recombining it back together.

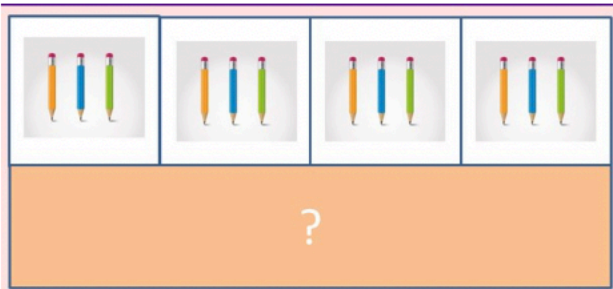


Counting in multi-  
ples of 2, 3, 4, 5, 10  
from 0  
(repeated addition)

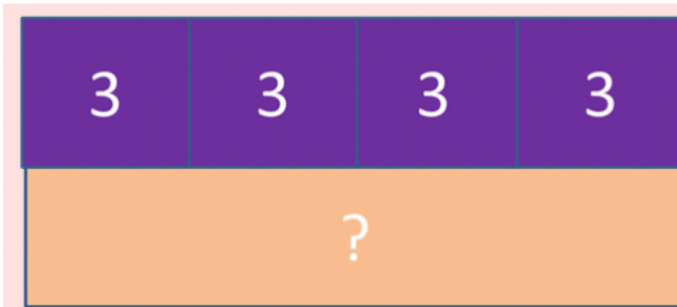
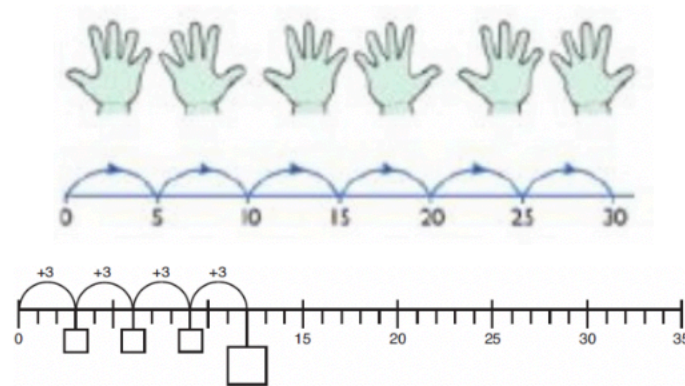
Count the groups as children are skip  
counting, children may use their fin-  
gers as they are skip counting. Use bar  
models.



$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$$



Number lines, counting sticks and bar  
models should be used to show repre-  
sentation of counting in multiples.



Count in multiples of a number aloud.

Write sequences with multiples of  
numbers.

0, 2, 4, 6, 8, 10

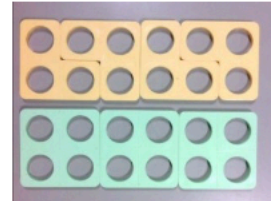
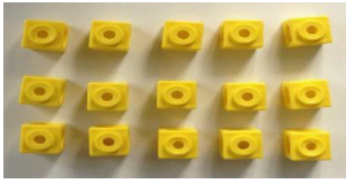
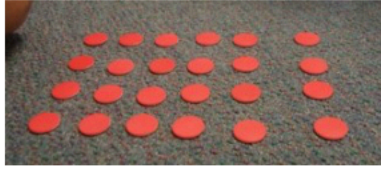
0, 3, 6, 9, 12, 15

0, 5, 10, 15, 20, 25, 30

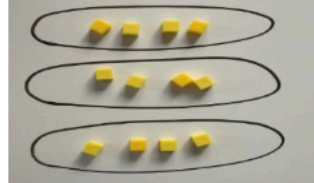
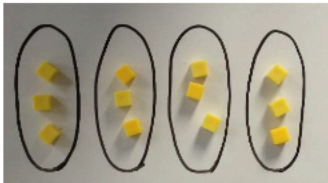
$$4 \times 3 = \square$$

Multiplication is commutative

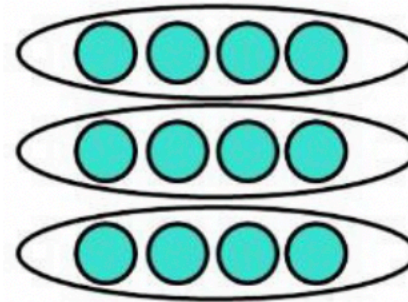
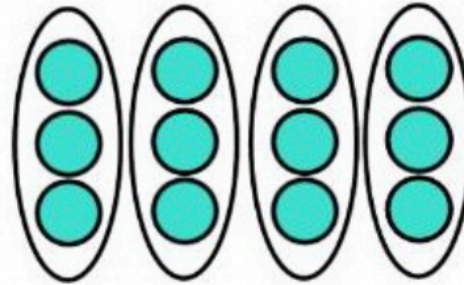
Create arrays using counters and cubes and Numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.



Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

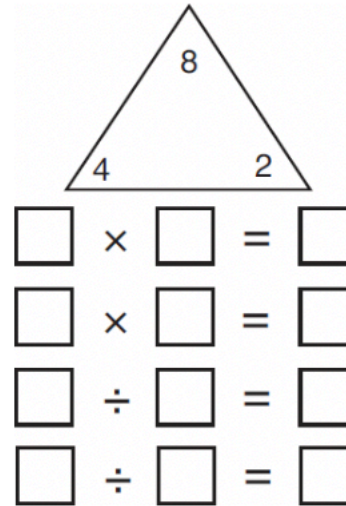
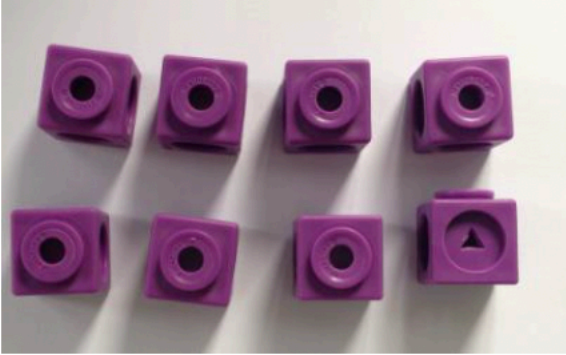
$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Using the Inverse

*This should be taught alongside division, so pupils learn how they work alongside each other.*



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

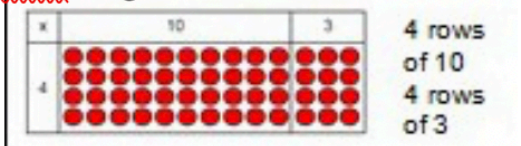
$$4 = 8 \div 2$$

Show all 8 related fact family sentences.

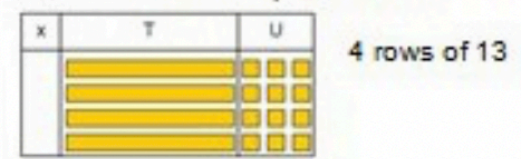


Grid method

Show the links with arrays to first introduce the grid method.



Move onto base ten to move towards a more compact method.

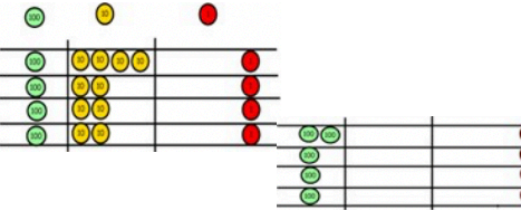


Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows



Fill each row with 126

Add up each column, starting with the ones making any exchanges needed

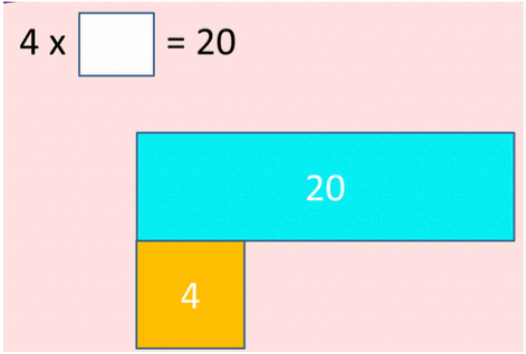


Then you have your answer.

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.

Bar model are used to explore missing numbers



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

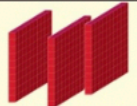
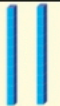

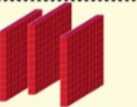


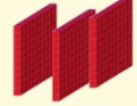
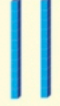

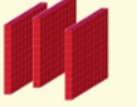


$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

## Column multiplication

Children can continue to be supported by place value counters at the stage of multipli-cation. This initially done where there is no regrouping.  $321 \times 2 = 642$

Hundreds	Tens	Ones
		
		
		
		

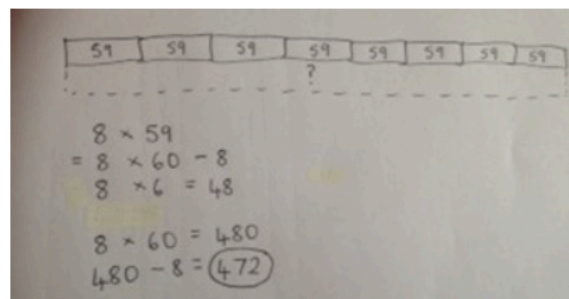
It is im-portant at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside

$\times$	300	20	7
4	1200	80	28



The grid method my be used to show how this relates to a formal written method.



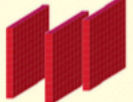


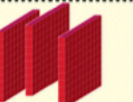


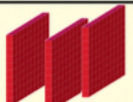


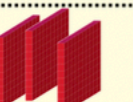
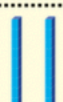

Bar modelling and number lines can support learners when solving problems with multipli-cation alongside the formal written methods.



$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 28 \\
 80 \\
 \hline
 1200 \\
 \hline
 1308
 \end{array}$$

This may lead to a compact method.

Column Multiplication for  
3 and 4 digits x 1 digit.

Hundreds	Tens	Ones
		
		
		
		

It is im-  
portant at  
this stage  
that they  
always  
multiply  
the ones  
first.

Children can continue to be supported by  
place value counters at the stage of multipli-  
cation. This initially done where there is no  
regrouping.  $321 \times 2 = 642$

x	300	20	7
4	1200	80	28



$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 28 \\
 80 \\
 1200 \\
 \hline
 1308
 \end{array}$$

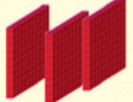


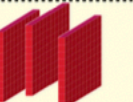


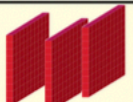


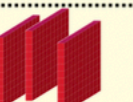
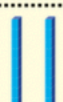



	3	2	7
x			4
	1	3	0
		1	2

This will lead to  
a compact  
method.



Column Multiplication for  
3 and 4 digits x 1 digit.

Hundreds	Tens	Ones
		
		
		
		

It is im-  
portant at  
this stage  
that they  
always  
multiply  
the ones  
first.

Children can continue to be supported by  
place value counters at the stage of multipli-  
cation. This initially done where there is no  
regrouping.  $321 \times 2 = 642$

x	300	20	7
4	1200	80	28



$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 28 \\
 80 \\
 1200 \\
 \hline
 1308
 \end{array}$$

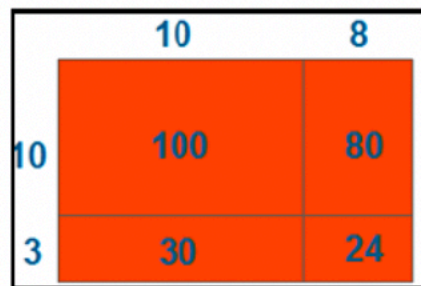


	3	2	7
x			4
	1	3	0
		1	2

This will lead to  
a compact  
method.

## Column multiplication

Manipulatives may still be used with the corresponding long multiplication modelled alongside.



		1	8	
	×	1	3	
		5	4	
		2		
	1	8	0	
	2	3	4	

18 x 3 on the first row

(8 x 3 = 24, carrying the 2 for 20, then 1 x 3)

18 x 10 on the 2nd row. Show

multiplying by 10 by putting zero in units first

	1	2	3	4	
×			1	6	
	7	4	0	4	(1234 × 6)
	1	2	3	4	0 (1234 × 10)
	1	9	7	4	4

Continue to use bar modelling to support problem solving

Multiplying decimals  
up to 2 decimal places  
by a single digit.

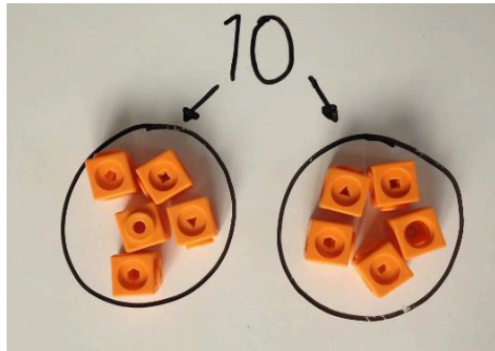
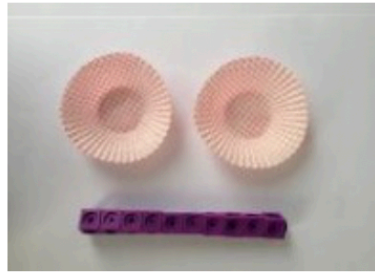
Remind children that the single digit belongs  
in the units column. Line up the decimal  
points in the question and the answer.

$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$$

# Division

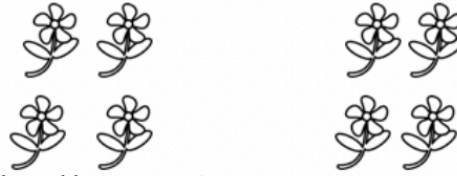
Division as sharing

Use Gordon ITPs for modelling



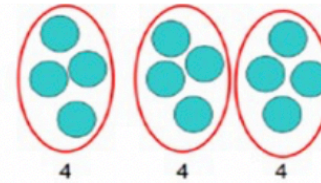
I have 10 cubes, can you share them equally in 2 groups?

Children use pictures or shapes to share quantities.



8 shared between 2 is 4

Sharing:

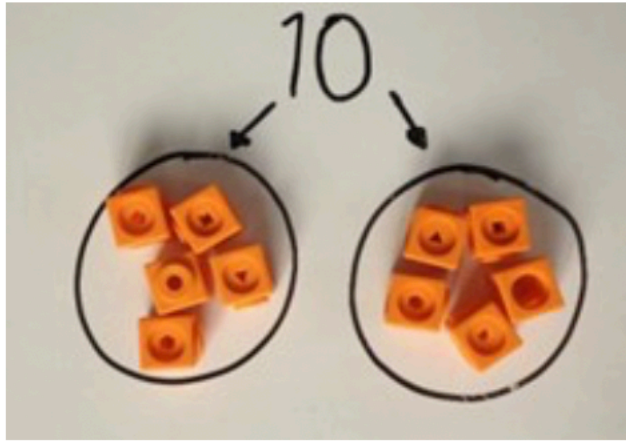


12 shared between 3 is 4

12 shared between 3 is

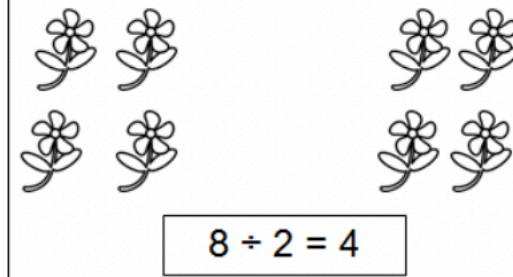
4

## Division as sharing

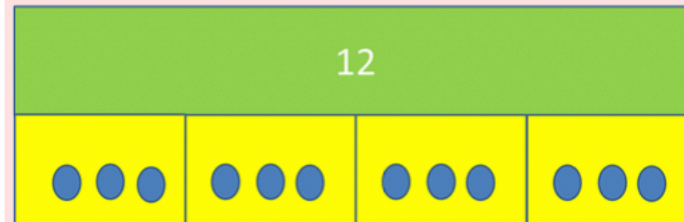


I have 10 cubes, can you share them equally in 2 groups?

Children use pictures or shapes to share quantities.



Children use bar modelling to show and support understanding.



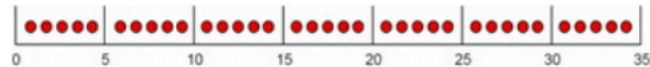
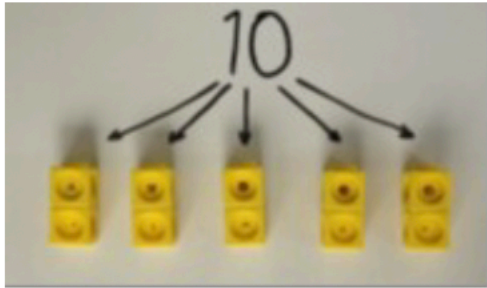
$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

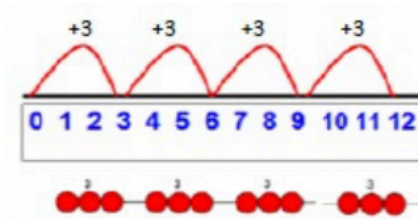
## Division as grouping

Divide quantities into equal groups.

Use cubes, counters, objects or place value counters to aid understanding.

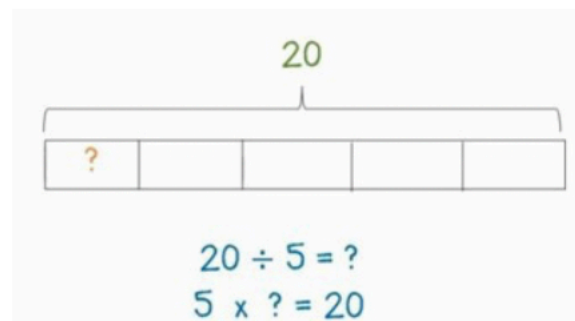


Use number lines for grouping



$$12 \div 3 = 4$$

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



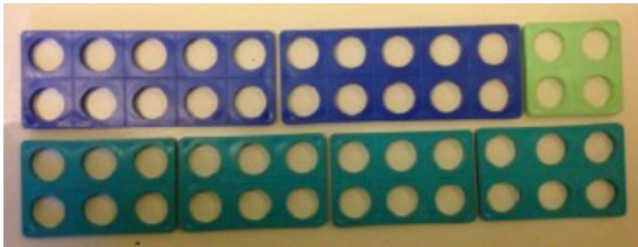
$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?



## Division as grouping

Use cubes, counters, objects or place value counters to aid understanding.

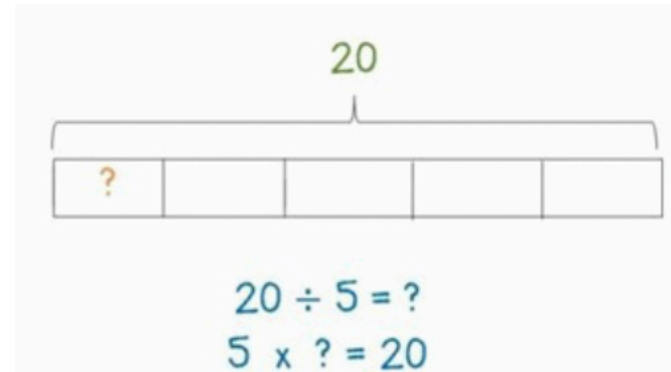


24 divided into groups of 6 = 4

$$96 \div 3 = 32$$



Continue to use bar modelling to aid solving division problems.

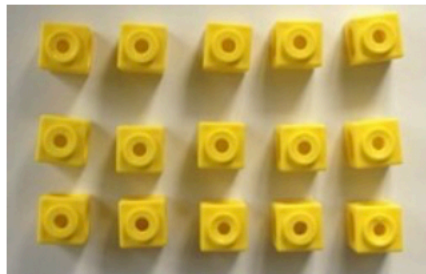


How many groups of 6 in  
24?

$$24 \div 6 = 4$$



## Division with arrays

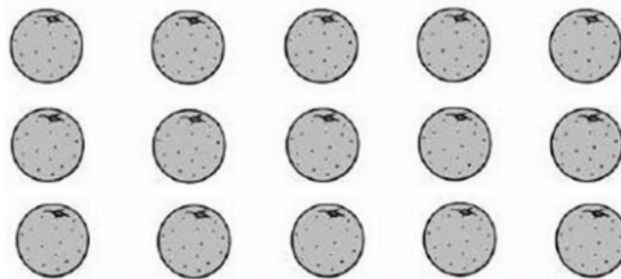


Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg  $15 \div 3 = 5$      $5 \times 3 = 15$

$15 \div 5 = 3$      $3 \times 5 = 15$

Draw an array and use lines to split the array into groups to make multiplication and division sentences



Find the inverse of multiplication and division sentences by creating eight linking number sentences.

$7 \times 4 = 28$

$4 \times 7 = 28$

$28 \div 7 = 4$

$28 \div 4 = 7$

$28 = 7 \times 4$

$28 = 4 \times 7$

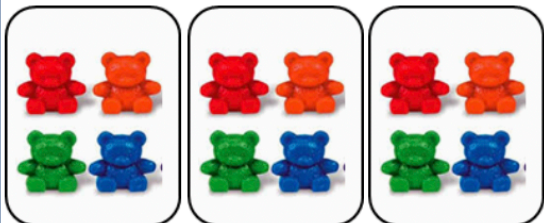
$4 = 28 \div 7$

$7 = 28 \div 4$

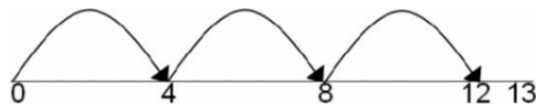
Division with remainders.

$14 \div 3 =$

Divide objects between groups and see how much is left over



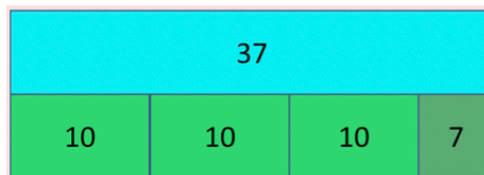
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



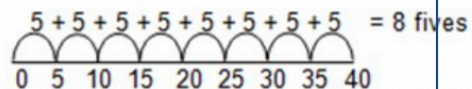
Use bar models to show division with remainders.



Example without remainder:

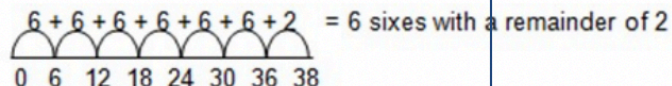
$40 \div 5$

Ask "How many 5s in 40?"



Example with remainder:

$38 \div 6$



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Complete written divisions and show the remainder using r.

$$\begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER} & 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array}$$

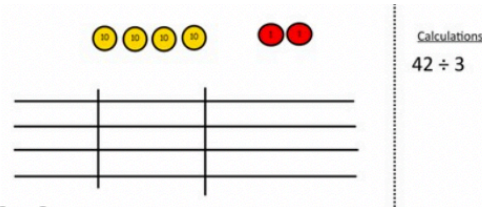
Divide at least 3 digit numbers by 1 digit.

## Short Division

$$96 \div 3$$

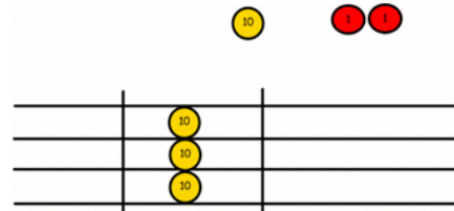


Use place value counters to divide using the bus stop method alongside

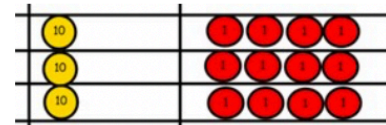


$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

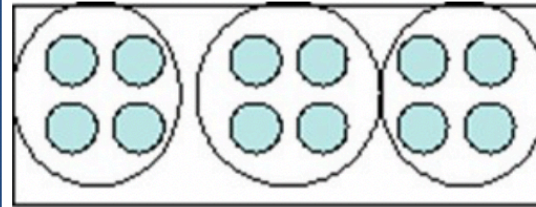


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 53029} \end{array}$$

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ( $3,200 \div 8 = 400$ )

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subtract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subtract. This finds us the remainder of 1.

Check:  $4 \times 402 + 1 = 1,609$

# Long Division

Show

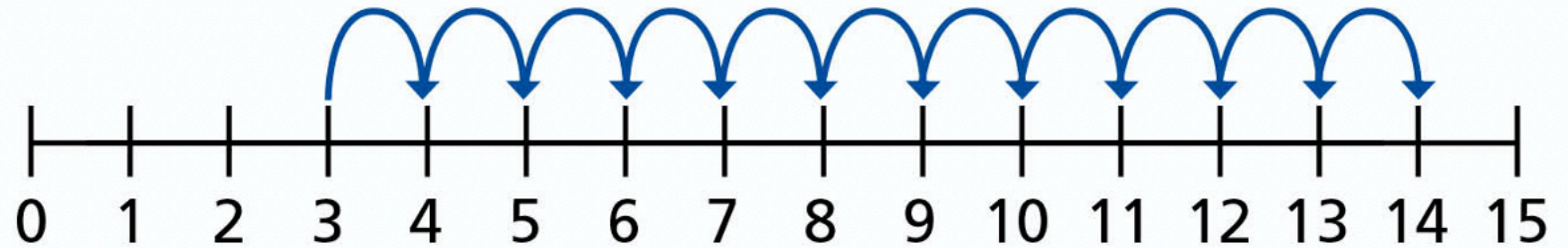
What does this look like in lessons?



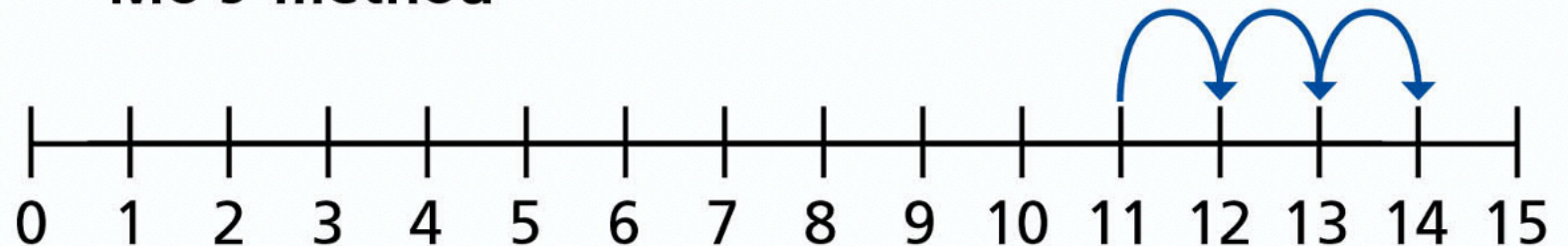
# Year 1

- 3** Ron and Mo are working out  $3 + 11$  on a number line.

**Ron's method**



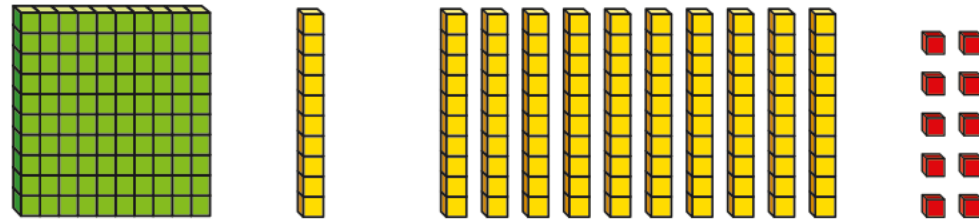
**Mo's method**



What is the same and what is different?

# Year 2

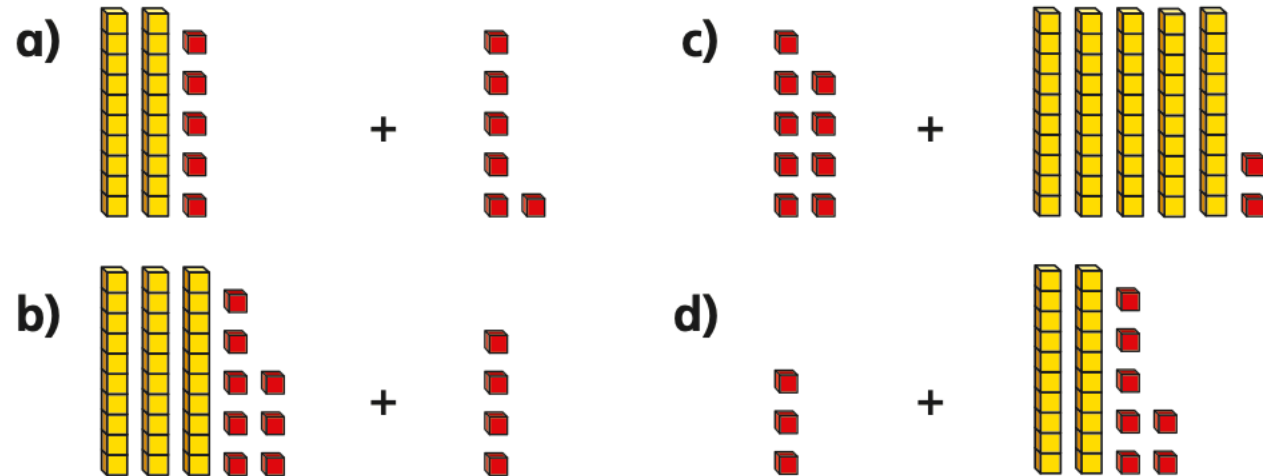
4 Which two representations show 10?



What is the same about the two representations?

What is different?

5 Work out the additions.



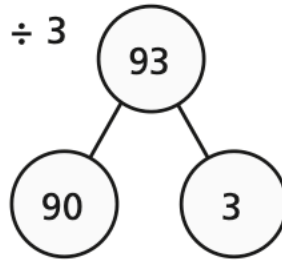


# Year 3

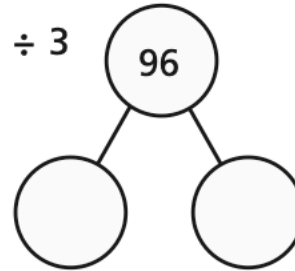
5

Work out the divisions.

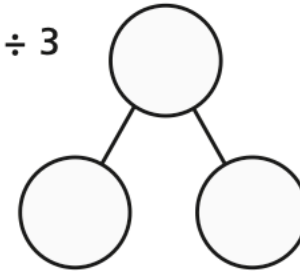
a)  $93 \div 3$



$96 \div 3$



$99 \div 3$



b)  $82 \div 2$      $84 \div 2$      $86 \div 2$

What do you notice?

6



88 can be  
divided equally by 2  
and by 4

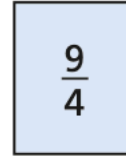
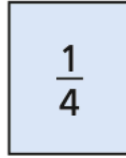
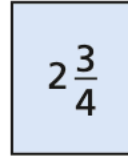
Do you agree with Annie?

Explain why.

Can Annie divide 88 equally by any other 1-digit numbers?

# Year 4

- 3 Match the numbers with a difference of  $\frac{3}{4}$



- 4 Aisha has 4 pies.



- a) Aisha gives  $\frac{5}{8}$  of a pie to Mo.

How many pies does Aisha have left?

- b) Aisha then gives 2 pies to Jack.

Calculate the difference between how much pie Aisha now has and how much pie Mo has.

# Year 5

- 6 Mr Hall has written these subtractions on the board.

$$45,541 - 25,865$$

**Rosie's workings**

$$\begin{array}{r} 2\ 5\ 8\ 6\ 5 \\ -\ 4\ 5\ 5\ 4\ 1 \\ \hline 2\ 0\ 3\ 2\ 4 \end{array}$$

$$68,945 - 34,758$$

**Whitney's workings**

$$\begin{array}{r} 6\ 8\ 9\ 4\ 5 \\ -\ 3\ 4\ 7\ 5\ 8 \\ \hline 3\ 4\ 2\ 1\ 3 \end{array}$$

Explain the mistakes that Rosie and Whitney have made.

- 7 Complete the subtractions.

a)  $10,004 - 9,995$

b)  $10,000 - 6,727$

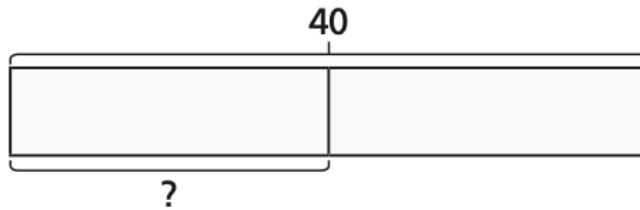
c)  $15,923 - 9,998$

How did you work this out?

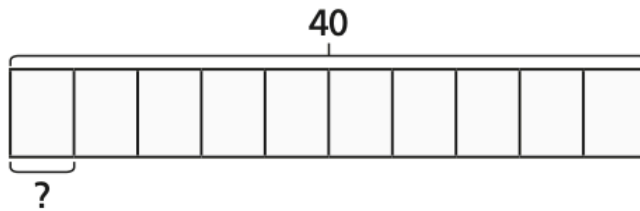
Is there another method you could use?

# Year 6

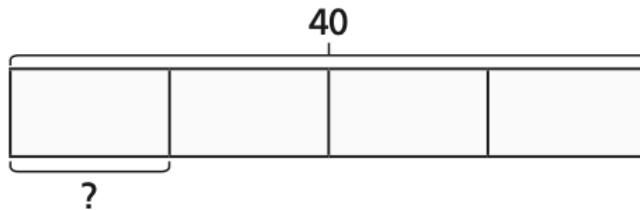
- 2 Match each bar model to the statement it represents.



10% of 40



25% of 40



50% of 40

Compare answers with a partner.



# Why bar models?

problems.

**I** Work out the missing values.

(a)

46	
26	20

(b)

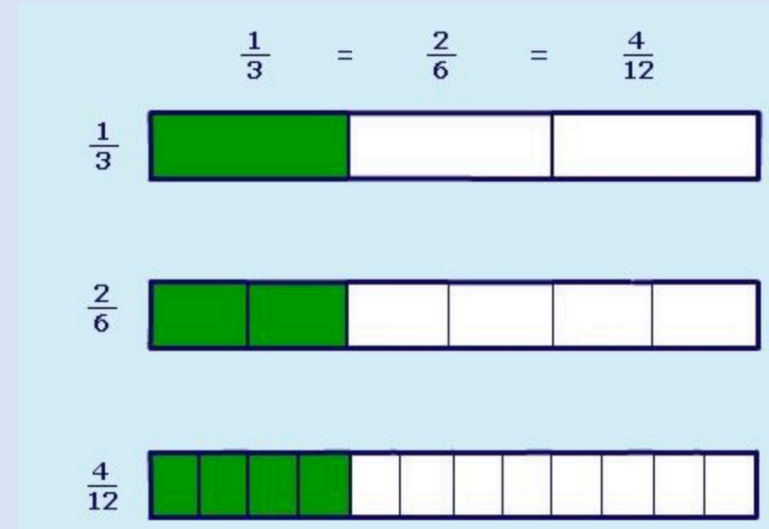
60	
12	

48

(c)

38	
28	

66
----



A barrel contains 175 litres of water.



2 buckets of water are poured into the barrel.



There is now 265 litres of water in the barrel.  
How much water could have been in each bucket?  
How many different answers can you find?

**Jess has 80 stickers.**

**Liam has 12 stickers.**

**Jess gives Liam 25 stickers.**

**How many more stickers than Liam does  
Jess have now?**

**Mo has 4 times as many sweets as Gill.**

**Mo eats  $\frac{3}{4}$  of his sweets.**

**He gives Gill  $\frac{1}{3}$  of the sweets he has left.**

**Mo has 18 sweets left.**

**How many sweets does Gill have now?**



**In a sale there is 10% off bikes.**

**Hannah buys a new bike in the sale.**

**She pays £189**

**How much did the bike cost before the sale?**

# Key messages to take home

- Progression in understanding
- Use of manipulatives to help children gain understanding
- Bigger numbers do not equal mastery and depth of understanding
- Mastery means being able to be fluent/ reason and problem solve